Hicks Decomposition

The Hicks decomposition refers to breaking down the effect that a price change has on the marshallian demand. When the price of a good increases, two effects arise that end up modifying the demand. On one hand, the individual now has less purchasing power than before, and on the other, the relative price ratio now changes. Based on the interaction of these two effects, goods can be classified into several categories:

- **Normal Good:** Its demand increases when the consumer's income increases and decreases when the income decreases. Here, the income effect reinforces the substitution effect.
- **Inferior Good:** Its demand decreases when the consumer's income increases.
- **Giffen Good:** A special case of the inferior good. Here, a price increase leads to an increase in demand, despite the substitution effect, because the income effect more than compensates for the substitution effect.
- **Typical or Ordinary Good:** In most cases, a price increase results in a decrease in demand.

On the other hand, the Marshallian and Hicksian demands represent two different approaches to depicting consumer preferences and their behavior in the face of price and income changes.

- **Marshallian Demand:** This demand represents how the quantity demanded of a good varies with changes in its price, keeping income constant. It is the demand we typically see on a supply and demand graph and reflects both the substitution effect and the income effect. It arises from maximizing utility subject to a budget constraint.
- **Hicksian Demand (or compensated demand):** This demand shows how the quantity demanded of a good changes when its price changes, but ensuring a constant utility level. In this way, it only reflects the substitution effect, eliminating the income effect. It is a theoretical tool that helps decompose and understand consumer behavior by isolating the substitution effect from the income effect.

Hicks decomposition and Slutsky equation

The Hicks decomposition can be represented as:

$$\Delta q = \Delta q_s + \Delta q_r \tag{1}$$

Where:

Δq is the total change in the quantity demanded (i.e., $q_1 - q_0$).
Δq_s is the change in the quantity demanded due to the substitution effect (i.e., $q_s - q_0$).
Δq_r is the change in the quantity demanded due to the income effect (i.e., $q_1 - q_s$).

On the other hand, we have the Slutsky equation that helps us understand this decomposition in terms of Marshallian and Hicksian demands:

$$\frac{\partial x^M(p,m)}{\partial p_i} = \frac{\partial x^H(p,u)}{\partial p_i} - x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m} \tag{2}$$

Where:

$\frac{\partial x^M(p,m)}{\partial p_i}$ is the change in the Marshallian demand with respect to the price of good i.

$\frac{\partial x^H(p,u)}{\partial p_i}$ is the substitution effect.

$x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m}$ is the income effect.

Giffen Goods

In the case of Giffen goods, what happens is that the income effect goes in the opposite direction to the substitution effect. They are inferior goods that have a very strong income effect. Analytically, note that the derivative $\frac{\partial x^H(p,u)}{\partial p_i}$ is always negative, while $x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m}$ can be negative or positive. Then, for Giffen goods, what happens is:

$$\frac{\partial x^M(p,m)}{\partial p_i} = \frac{\partial x^H(p,u)}{\partial p_i} - x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m} > 0 \tag{3}$$

$$\frac{\partial x^H(p,u)}{\partial p_i} > x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m} \tag{4}$$

And for this to occur, since the derivative of the Hicksian demand is negative, the term $\frac{\partial x^M(p,m)}{\partial m}$ must have a negative sign (for the good to be inferior) and also $x^M(p,m) \cdot \frac{\partial x^M(p,m)}{\partial m}$ must be a very high absolute number (so that the income effect exceeds the substitution effect).

Graphical example of a normal and typical good

In the following graph, the effect of a price reduction of good x_1 is shown, which generates an increase in the demand for said good. Points A and C represent the initial and final situations, respectively. Point B, however, is a hypothetical situation where the individual has less income (so as to have the initial utility) but the price ratio is the same as in the final outcome. This is done in order to decompose the income and substitution effects. From point A to B, the demand increase is due to the substitution effect, while from point B to C, the demand increase is due to the income effect.